

1. Zderivujte funkce:

$$f(x) = 2x^3 + 5x^7 - x^8 \quad [6x^2 + 35x^6 - 8x^7]$$

$$f(x) = \arcsin x + \operatorname{arctg} x, x \in (-1,1) \quad \left[\frac{1}{\sqrt{1-x^2}} + \frac{1}{1+x^2} \right]$$

$$f(x) = \sqrt[3]{x^2} - \frac{2}{\sqrt{x}}, x > 0 \quad \left[\frac{2}{3\sqrt[3]{x}} + \frac{1}{\sqrt{x^3}} \right]$$

$$f(x) = 2^x - \operatorname{cotg} x + \frac{1}{\sqrt[5]{x^3}}, x \in (0, \pi) \quad \left[2^x \ln 2 + \frac{1}{\sin^2 x} - \frac{3}{5x^{\frac{8}{5}}} \right]$$

$$f(x) = x^2 \operatorname{arctg} x \quad \left[2x \operatorname{arctg} x + x^2 \frac{1}{1+x^2} \right]$$

$$f(x) = e^x 3^x \quad [e^x 3^x + e^x 3^x \ln 3]$$

$$f(x) = x^2(x^3 - 2x + 1) \quad [x(5x^3 - 6x + 2)]$$

$$f(x) = x \ln x, \quad x > 0 \quad [1 + \ln x]$$

$$f(x) = \frac{\sqrt{x}}{\ln x} \quad \left[\frac{\ln x - 2}{2\sqrt{x} \ln^2 x} \right]$$

$$f(x) = \frac{1 + e^x}{1 - e^x}, x \neq 0 \quad \left[\frac{2e^x}{(1 - e^x)^2} \right]$$

$$f(x) = \frac{x}{1 - \cos x}, x \in (0, 2\pi) \quad \left[\frac{1 - \cos x - x \sin x}{(1 - \cos x)^2} \right]$$

$$f(x) = \frac{x^2 - 1}{x^2 + 1} \quad \left[\frac{4x}{(x^2 + 1)^2} \right]$$

$$f(x) = \sin^3(3x + 5) \quad [9 \sin^2(3x + 5) \cos(3x + 5)]$$

$$f(x) = \sqrt{1 + \ln^2 x} \quad \left[\frac{\ln x}{x\sqrt{1 + \ln^2 x}} \right]$$

$$f(x) = \operatorname{arctg}^2 x^2 \quad \left[\frac{4x \operatorname{arctg} x^2}{1 + x^4} \right]$$

$$f(x) = \sqrt{x^2 + 1} \quad \left[\frac{x}{\sqrt{x^2 + 1}} \right]$$

2. Vypočtěte derivaci funkce $f(x)$ v bodě x_0

$$f(x) = e^{\sqrt{x+1}}, x_0 = 0 \quad \left[\frac{1}{2} e \right]$$

3. Vypočítejte druhé derivace

$$f(x) = \frac{\cos x}{x}$$

$$f(x) = \frac{1}{x^2}$$

$$\left[f''(x) = \frac{2x \sin x - (x^2 - 2) \cos x}{x^3} \right]$$

$$\left[f''(x) = \frac{6}{x^4} \right]$$

4. Vypočítejte první, druhou a třetí derivaci funkce $f(x) = \sqrt{x}$ v bodě $x_0=1$

$$\left[f'(1) = \frac{1}{2}; f''(1) = -\frac{1}{4}; f'''(1) = \frac{3}{8} \right]$$

5. Vypočítejte limity funkcí s užitím l'Hospitalovým pravidlem

a) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^3 + x^2 - 6x} =$

b) $\lim_{x \rightarrow \infty} \frac{\ln x}{x} =$

c) $\lim_{x \rightarrow \infty} \frac{x^3 - 1}{x^4 + 1} =$

d) $\lim_{x \rightarrow \infty} e^{-x} x^2 =$

[a) 0; b) 0; c) 0; d) 0]

6. Vypočítejte parciální derivace podle x a y

$$f(x) = e^{x^2+y^2}$$

$$\left[\frac{\partial f}{\partial x} = e^{x^2+y^2} \cdot 2x; \frac{\partial f}{\partial y} = e^{x^2+y^2} \cdot 2y \right]$$

$$f(x) = \operatorname{arctg} \frac{x}{y}$$

$$\left[\frac{\partial f}{\partial x} = \frac{y}{x^2 + y^2}; \frac{\partial f}{\partial y} = -\frac{x}{x^2 + y^2} \right]$$